Implementation of the Ensemble Adjustment Kalman Filter in a Global NWP Model for Data Assimilation

Shaoqing Zhang and Jeffrey L. Anderson

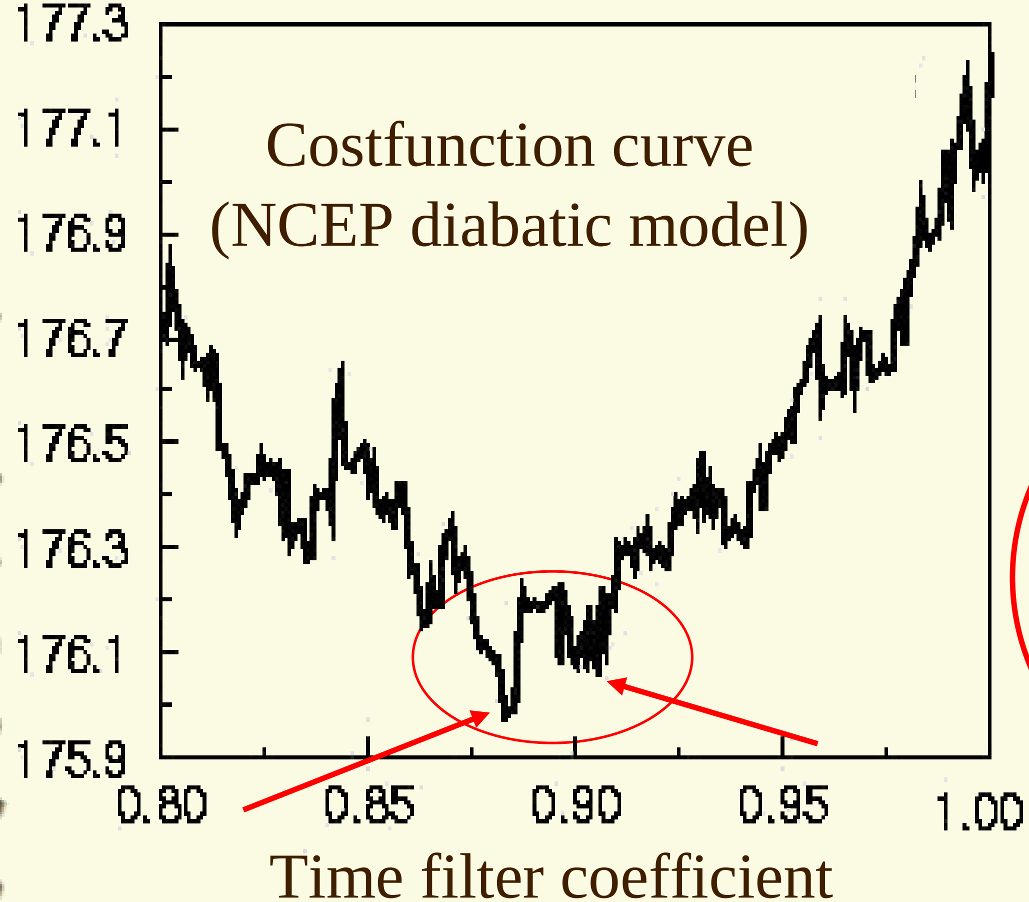
Geophysical Fluid Dynamics Laboratory

Princeton, New Jersey

Primary motivation:

Variational Approach: Minimizing an error function (costfunction) between model trajectory and obs for optimizing ICs.

Question: **Where should the solution be?**



- ✓ A single estimate is not a good representation of solution.
- ✓ An ensemble filter gives a set of estimates sampling the probability distribution of solution.

Keep in mind: Two major points

- ✓ Filtering may unify all modern Data Assimilation algorithms based on different approximation of error covariance evolution.

- ✓ An ensemble filter may produce the best estimate due to considering the nonlinear evolution of error covariance in time.

Outline

- ✓ Introduction.
  - What's Data Assimilation?
  - What's filtering? What good is filtering for Data Assimilation?
- ✓ Brief review of modern Data Assimilation algorithms.
  - Optimal Interpolation (OI) (Gandin 1963).
  - Variational approach (4DVAR) (LeDimet & Talagrand 1986).
  - Kalman-Bucy filter (Kalman and Bucy 1961).
  - Ensemble Kalman filter (Evensen 1994).
- ✓ Ensemble adjustment Kalman filter (EAKF) on a realistic NWP model.
  - Description of EAKF algorithm and its advantages.
  - Numerical results on a fully-parameterized NWP model.
    - Estimate of error covariances.
    - Examination of prognostic variables.
    - Examination of precipitation rate.
  - Numerical results of an ensemble OI experiment.
- ✓ Summary and Future work.

Introduction: What is Data Assimilation?

- Atmosphere/Ocean: ***The evolution has a probabilistic nature***
  - Modeling equation (stochastic):

$$d\mathbf{x}_t / dt = \underbrace{\mathbf{f}(\mathbf{x}_t, t)}_{\text{Deterministic model}} + \mathbf{G}(\mathbf{x}_t, t)\underbrace{\mathbf{w}_t}_{\text{White Gaussian process}}$$

  - Observations: Noisy and irregularly spaced.

- Data Assimilation: ***Use model dynamics to extract information from observations to reconstruct the structure of the system.***

Introduction: What is filtering? What good is filtering for DA?

- Filtering: ***Use observational probability density function (PDF) to modify the prior PDF from model.***

- Good for Data Assimilation:
  - Directly address the probabilistic nature of the problem.
  - A single estimate has the most likelihood (linear/stationary filter).
  - A set of estimates are an ensemble of possible states (ensemble filter).

Challenge: Evaluate PDFs and compute their product

- ✓ Bayes' rule underlies filtering computations:

$$p(\mathbf{x}|\mathbf{Y}_t) = p(\mathbf{y}^0)\, p(\mathbf{x}|\mathbf{Y}_{t-}) \,/\, p(\mathbf{y}^0|\mathbf{Y}_{t-})$$

Modified PDF of **x** given all obs up to t, $\mathbf{Y}_t$

PDF of current obs (being assimilated, $\mathbf{y}^0$)

Prior PDF of **x**

Normalization

  - How to evaluate these probability distributions?
  - How to compute the product of these PDFs?
  - How to produce estimates of **x** using $p(\mathbf{x}|\mathbf{Y}_t)$?
- ✓ Different approximations form a variety of modern DA algorithms.

Review of DA algorithms: OI, 4DVAR and Kalman filter

- ✓ OI: A minimum variance estimate using a constant prior covariance matrix (unchanged in time).
  - Gandin (1963): minimizing analysis err variance of a linear combination of obs increments (least square).
  - Lorenc (1986): minimizing a costfunction (variational).
- ✓ 4DVAR (LeDimet & Talagrand 1986): A minimum variance estimate by minimizing a distance between model trajectory and obs using adjoint to derive the gradient under model's constraint.
- ✓ Kalman filter (Kalman and Bucy 1961): linear error dynamics + linear observations:

$$\mathbf{x}_a = \mathbf{x}_f + \mathbf{PH}^T(\mathbf{HPH}^T + \mathbf{R})^{-1}(\mathbf{y} - \mathbf{Hx}_f)$$

  - Jazwinski (1970): a maximum likelihood estimate (Kolmogorov's PDF eq.)
  - Miller (1991): a minimum variance estimate (solve a weak constraint variational problem)
- ✓ **Kalman filter and 4DVAR: Linear filter. OI: stationary filter.**

✓$\mathbf{x}_a$=analysis state.
✓$\mathbf{x}_f$=forecast state.
✓**y**=obs (var: **R**).
✓**H**=mapping operator from model to obs.
✓**P**=prior covariance matrix.

Review of DA algorithms: Ensemble Kalman filter (ENKF, Evensen 1994)

- Use a set of ensemble members to discretely represent the state's PDF to compute the sample statistics such as covariance and mean.
- Use the Kalman filter's analysis formulation to update each ensemble member.
- Perturbing observations using obs value and error variance (Houtekamer 1998) to form an obs distribution in order to carry out "approximating random samples of the product using the product of random samples" (Burgers et al. 1998).

$$\boxed{\mathbf{x}_a^i=\mathbf{x}_f^i+\mathbf{PH}^T(\mathbf{HPH}^T+\mathbf{R})^{-1}(\mathbf{y}^i - \mathbf{Hx}_f^i) \text{ for the i-}\textit{th}\text{ ens member}}$$

- **Advantage:** Consider the nonlinear time-evolution of error (prior) covariance matrix using ensemble.
- **Disadvantage:**
  - Perturbing observations introduce noise for analyses.
  - Sampling problems.

An ensemble adjustment Kalman filter (EAKF) (Anderson 2001): Algorithm

- ✓ Define a joint state/obs vector $\mathbf{z}=\{\mathbf{x}_t,\mathbf{h}(\mathbf{x},t)\}$, $\mathbf{h}$ gives the expected value of obs from model state.
- ✓ Use a set of prior ensemble members (advancing the model from the previous analysis) to compute prior cov matrix.

$$\Sigma^p = \begin{pmatrix} \sigma_g^2 & \mathrm{cov}_{go} \\ \mathrm{cov}_{go} & \sigma_o^2 \end{pmatrix}$$

g: model gridpoint
o: obs location

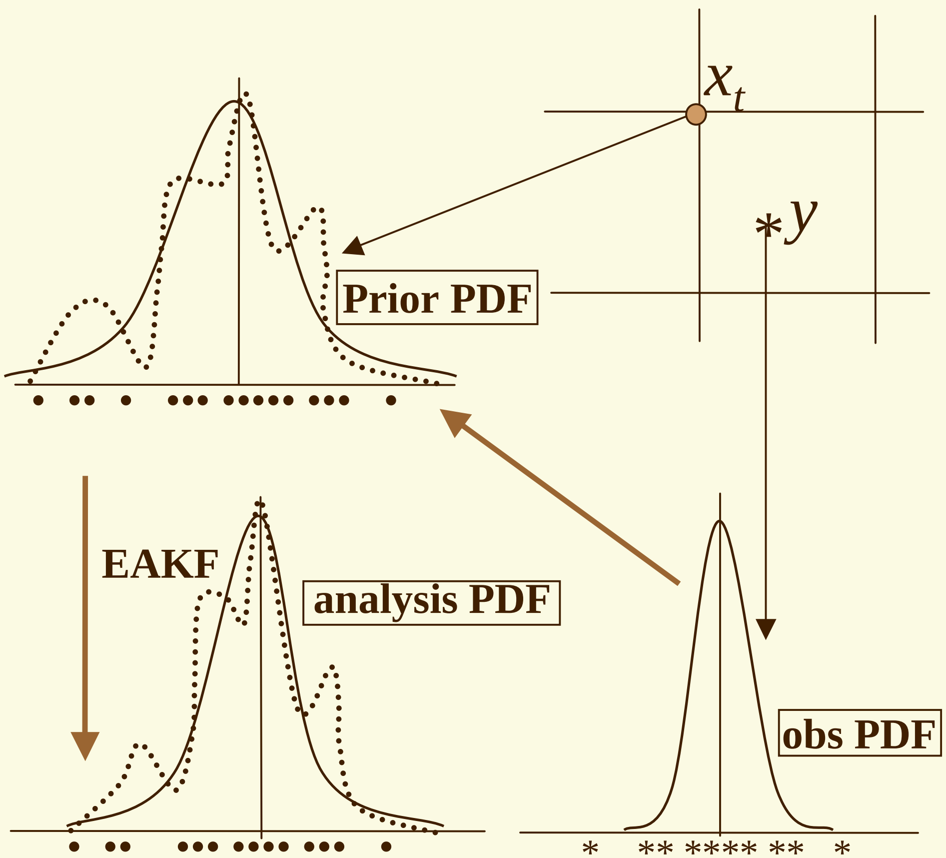
- ✓ Compute the product of two Gaussians (Gaussian convolution)

$$\begin{cases} \Sigma^u = [(\Sigma^p)^{-1} + \mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}]^{-1} \\ \overline{\mathbf{z}^u} = \Sigma^u[(\Sigma^p)^{-1}\overline{\mathbf{z}^p} + \mathbf{H}^T\mathbf{R}^{-1}\mathbf{y}^o] \end{cases}$$

- ✓ Update the ensemble members using a linear operator $\mathbf{A}$:

$$\begin{cases} z_i^u = \mathbf{A}^T(z_i^p - \overline{z^p}) + \overline{z^u} \\ \Sigma^u = \mathbf{A}\Sigma^p\mathbf{A}^T \end{cases}$$

An ensemble adjustment Kalman filter (EAKF) (Anderson 2001): Advantages



- The generated new ensemble maintains the non-Gaussian information of prior distribution.
- The algorithm does not require perturbed observations.
- The algorithm only processes 2x2 matrix, which just requires small storage and cheap computational cost.
- **So, the algorithm allows the ensemble filter to be applied to a realistic NWP model.**

Model: A fully-parameterized B-grid version of FMS at GFDL

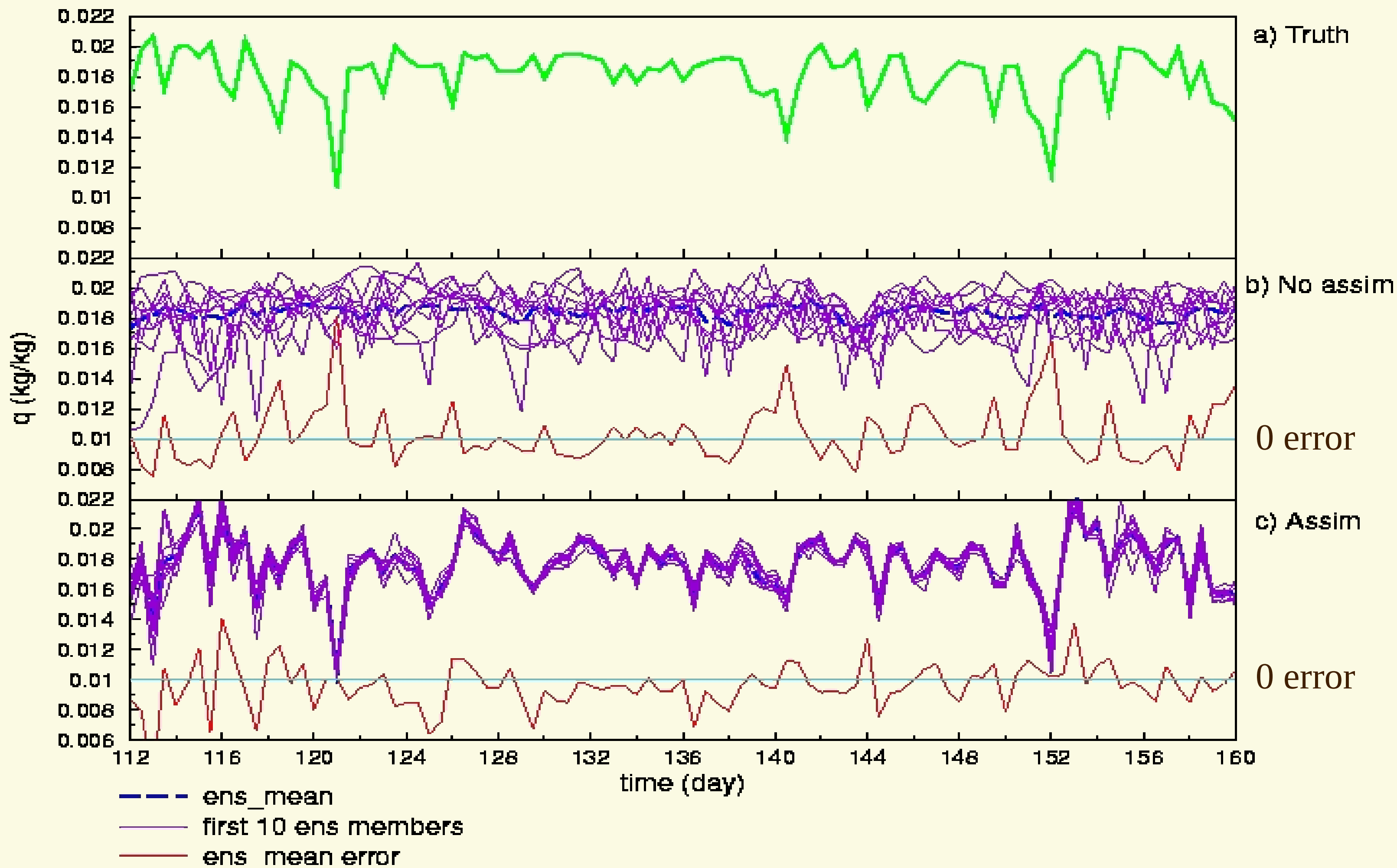
- ✓ Dynamical core: (Wyman 1996 from an early E-grid version)
  - A global B-grid configuration (Arakawa and Lamb 1977).
  - σ/p hybrid vertical coordinate.
  - Prognostic equations: Momentum (u & v), temperature (T), specific humidity (q) and surface pressure (p_s).
  - Resolution: 90 x 60 (4^o x 3^o) x 18 (N30L18).
- ✓ Parameterizations:
  - Long- and short-wave radiation (Fels and Schwarzkopf 1975; 1991).
  - Moist-convection adjustment (Manabe et al. 1965) and large-scale condensation.
  - Vertical turbulence (Mellor and Yamada 1982).
  - Gravity-wave drag (Pierrehumbert 1987).
  - Land processes (a simple ‘bucket’ hydrology).

Observing/assimilation simulation experiment: Perfect model study

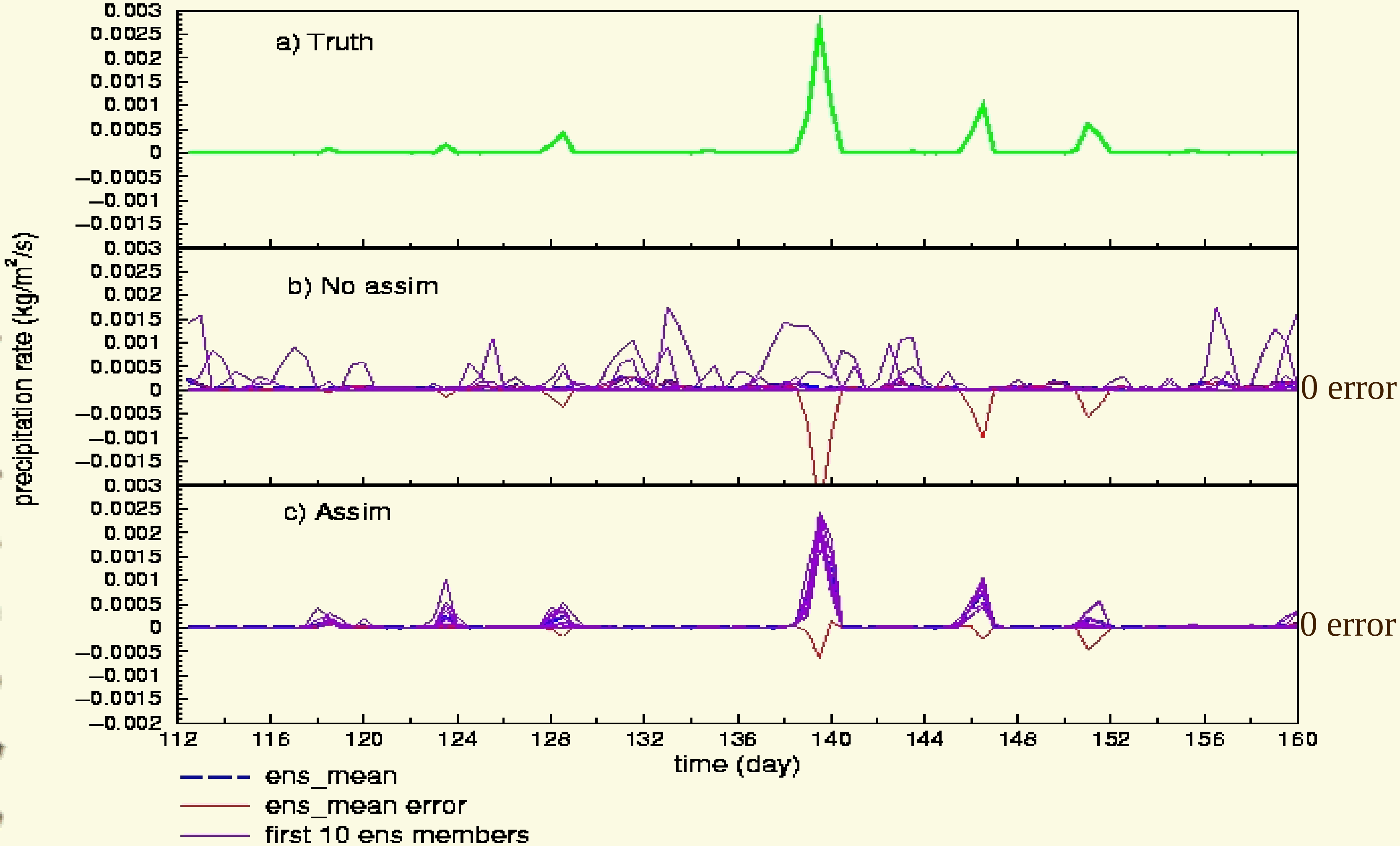
- ✓ Observational network: 600 randomly distributed locations (vertical columns) over a global domain, available every 12 hours.
- ✓ Observations: a control run + white noise error, N(0, σ).

  Observational error standard deviation (σ): 1 m/s (u & v), 1 k (T), 1 mb (P_s), 10^{-4} kg kg^{-1} (q) .
- ✓ Initial conditions: ECMWF re-analysis at 00UTC on 01/01/79.
- ✓ Ensemble ICs: re-analysis data set + Gaussian errors.
- ✓ A 10^o/cos(lat) x 10^o impact window and only obs of the same physical variable at the same level: each model gridpoint at mid latitude is impacted by about 40 obs.
- ✓ System is spun up for 72 days using 20 ensemble members.
- ✓ Another 365-day assim (day 73 to day 437) was conducted.
- ✓ The period of day 112 to day 160 is examined in detail.

Assimilation results: q time series at (176E, 0) at model level 3 (sigma=0.946)



Assimilation results: precipitation rate time series at (176E, 0)



An ensemble OI assimilation experiment

- Same observational network as used by EAKF experiment before.
- Ensemble initial conditions: the assimilation results of EAKF at day 72.
- EAKF's update formulation:

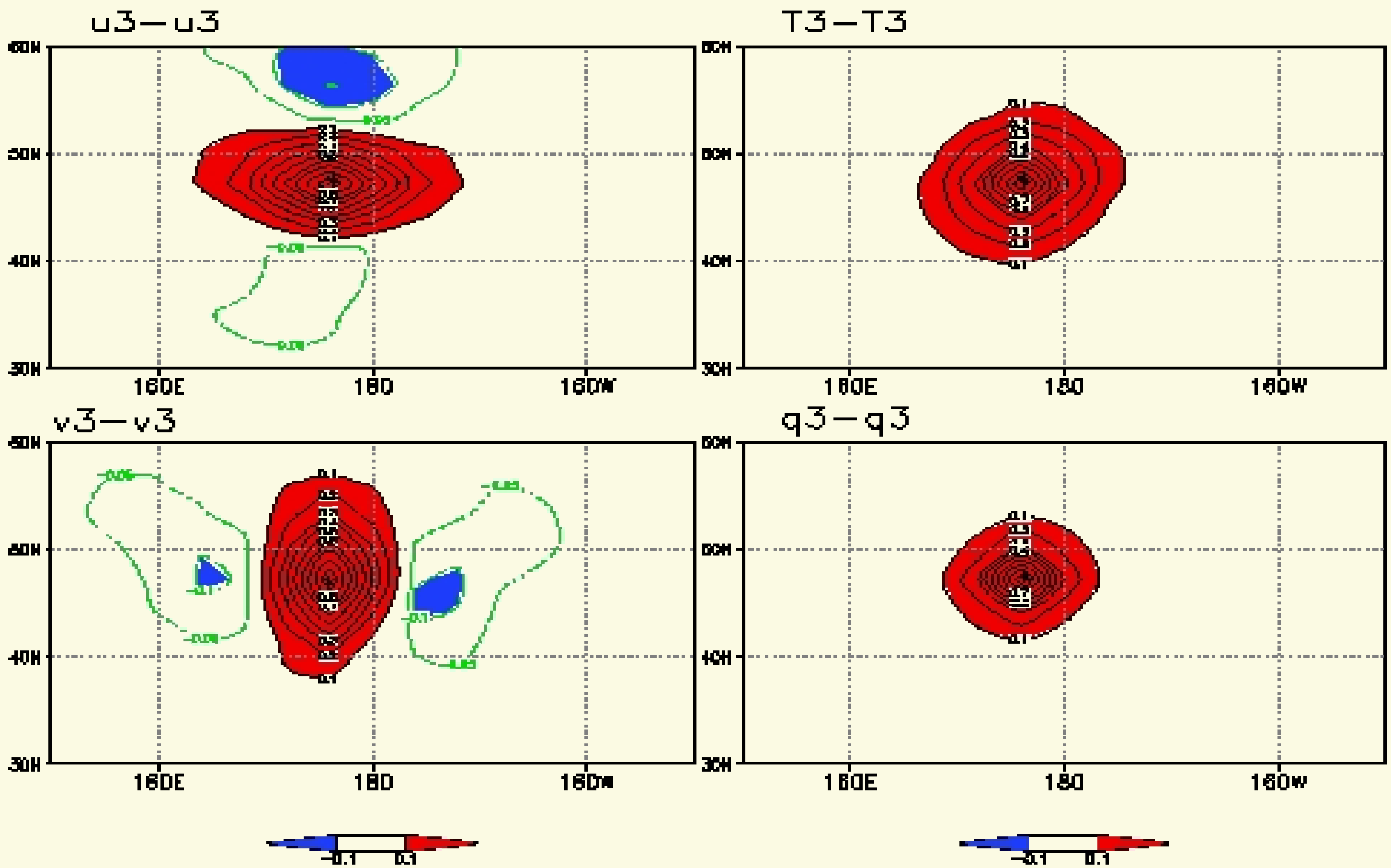
$$\begin{cases} \Sigma^u = [(\Sigma^p)^{-1} + \mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}]^{-1} \\ \overline{\mathbf{z}^u} = \Sigma^u[(\Sigma^p)^{-1}\overline{\mathbf{z}^p} + \mathbf{H}^T\mathbf{R}^{-1}\mathbf{y}^o] \end{cases}$$

$$\begin{cases} z_i^u = \mathbf{A}^T(z_i^p - \overline{z^p}) + \overline{z^u} \\ \Sigma^u = \mathbf{A}\Sigma^p\mathbf{A}^T \end{cases}$$

- A time-averaged (stationary) error covariance matrix
- Picking the period of day 112 to day 160 for exam and comparisons.

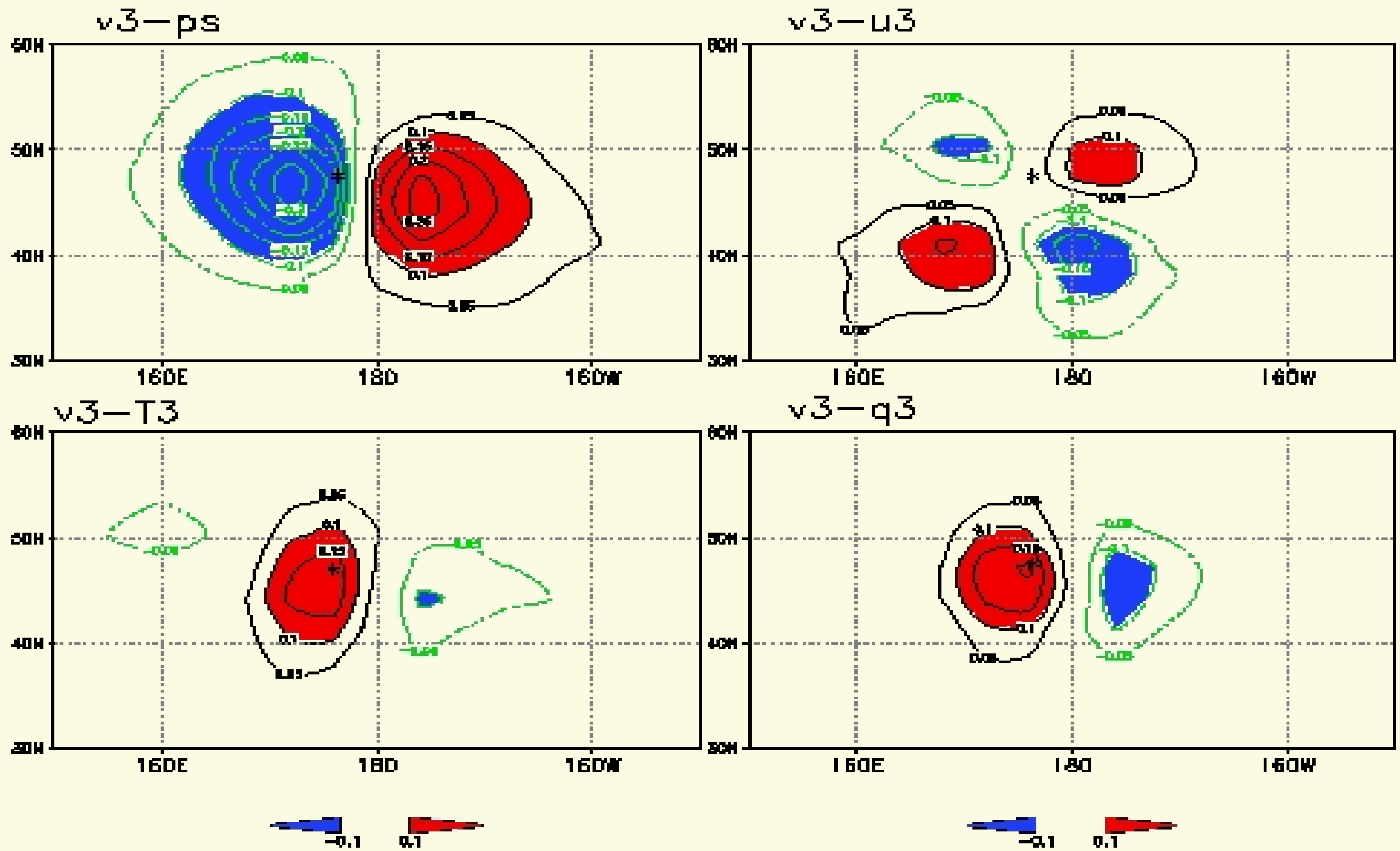
Estimate of stationary auto-covariance matrix: u, v, T and q at model level 3 (sigma=0.946)

- ✓Time-averaged over day 112 to day 160.
- ✓Zonally-averaged over all key points at the same latitude.

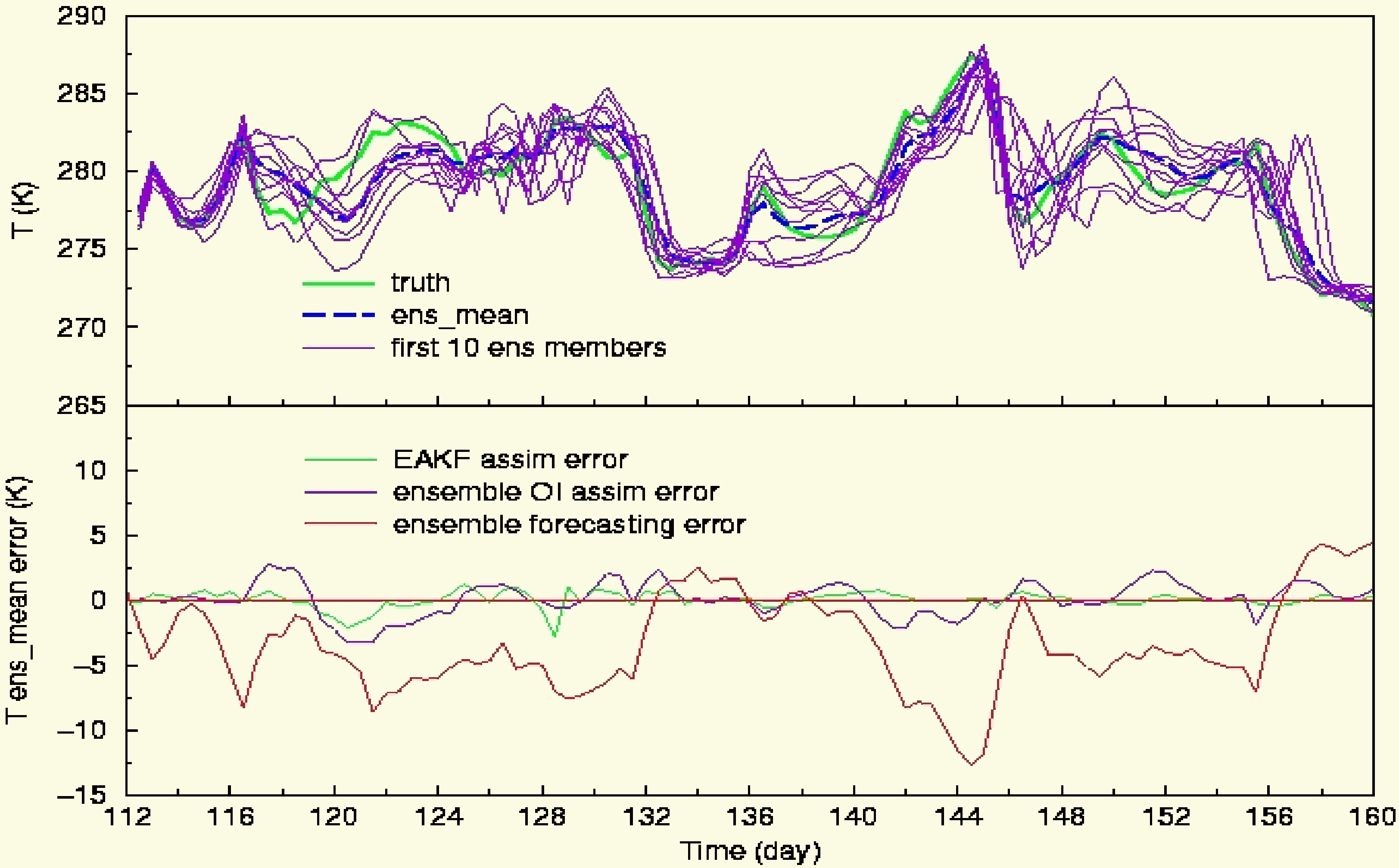


Estimate of stationary cross-covariance matrix v at model level 3 and ps, u, t and q

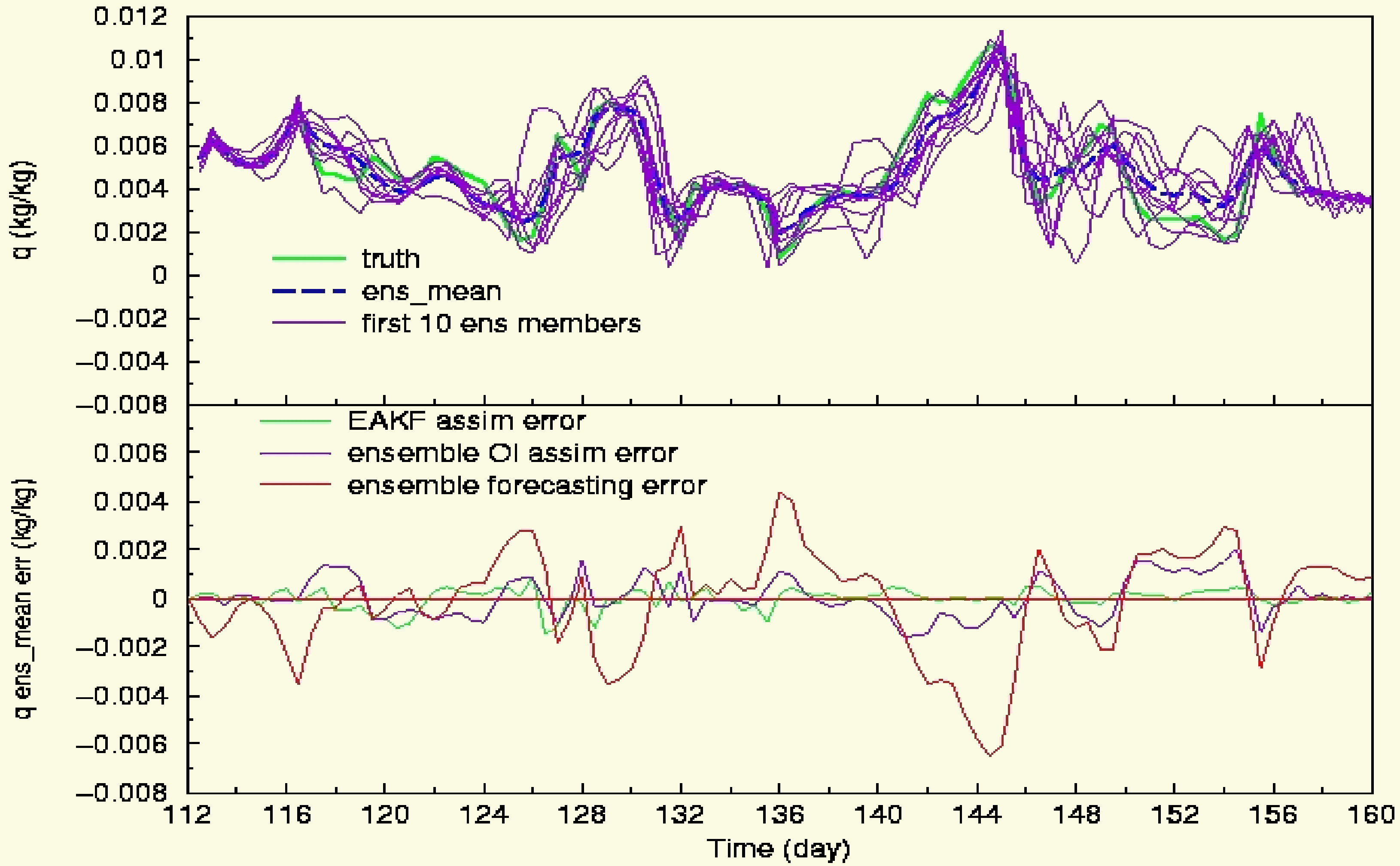
- ✓ Time-averaged over day 112 to day 160.
- ✓ Zonally-averaged over all key points at the same latitude.



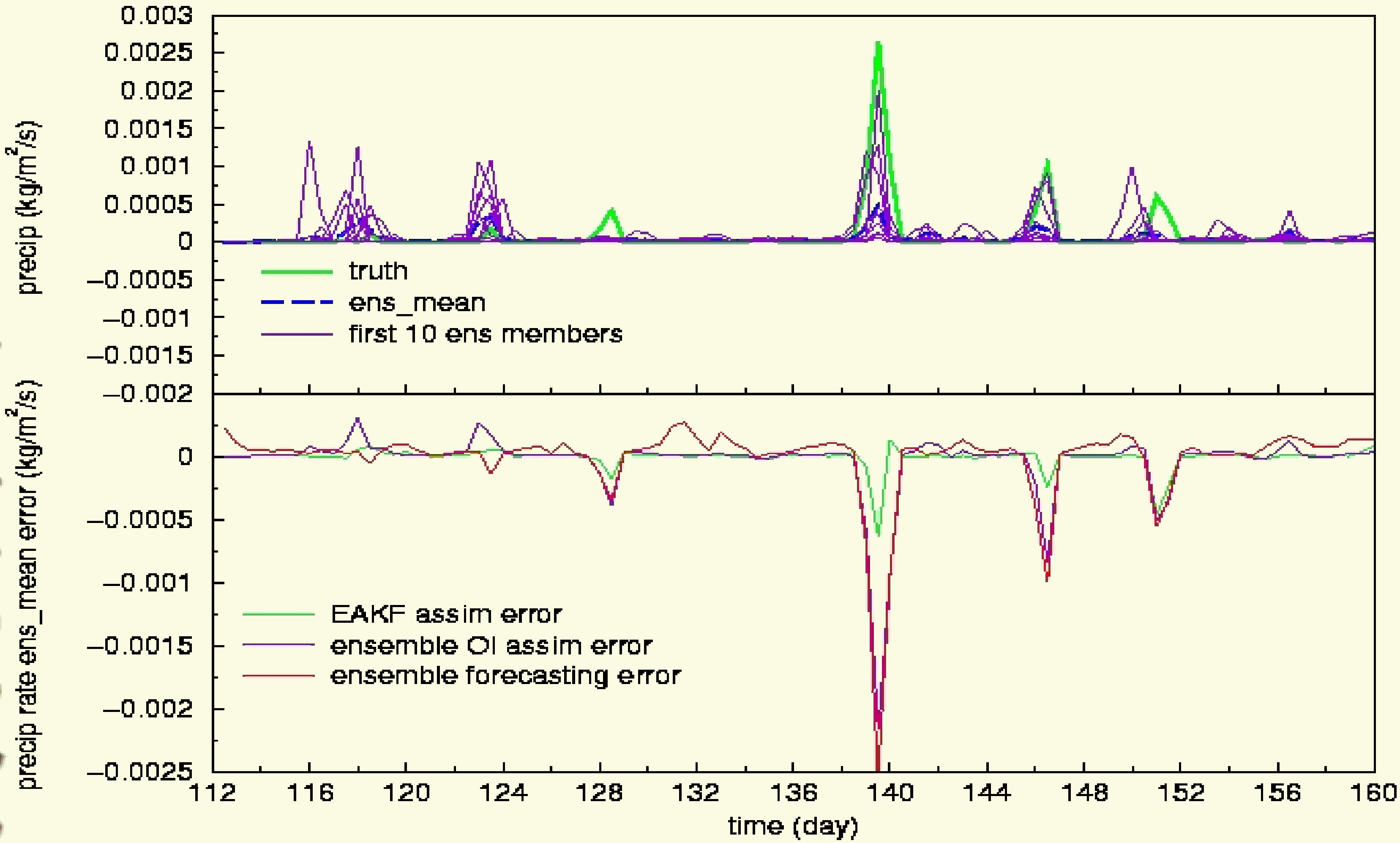
Ensemble OI assim results: time series of T (176E, 45N) at model level 3 (sigma=0.946)



Ensemble OI Assim results: time series of q (176E, 45N) at model level 3 (sigma=0.946)



Ensemble OI assim results: time series of precip rate (176E, 0)



Summary

- Filtering unifies the modern assimilation algorithms:
  - OI (3DVAR) is a stationary filter.
  - 4DVAR/Kalman filter is a linear filter.
  - Ensemble filter accounts the nonlinear time evolution of covariance matrix and therefore may produce the best assimilation results.
- With many advantages (maintaining the non-Gaussian characteristics of prior distribution, not perturbing obs and only processing 2x2 matrix), the ensemble adjustment Kalman filter (EAKF) is able to
  - Conveniently estimate a flow-dependent error covariance in a multi-variate system.
  - Efficiently assimilate observational data.
- Ensemble OI induced by EAKF using a stationary (flow-independent) error covariance has a certain capability to assimilate observational data, but the assimilation error is around three times more than EAKF that uses flow-dependent error covariance.

Future work

- ✓ The model's bias (imperfect model) and the quality control of observational data.
- ✓ Implement EAKF in ocean model (MOM4, for instance) and explore the new ocean data assimilation approach (first consider perfect model).
- ✓ Examine the characteristics of flow-independent/dependent error covariance using the ocean model to understand the model dynamics.
- ✓ Once the model's adjoint is available, a direct comparison of the assimilation results using EAKF and 4DVAR provides more insight about data assimilation philosophies.
- ✓ Explore a feasible unification of EAKF and variational approach by introducing the flow-dependent error covariance in 4DVAR to improve data assimilation technologies.
- ✓ Study the possibilities to improve OI algorithm using the estimated stationary error covariance matrix from the realistic model.

Thanks

- ✓ Tony Rosati and Matt Horrison for discussions and help.
- ✓ FMS modelers for their hard work on developing the model and providing the accessible model system.
- ✓ E-group and GFDL for nice work environment.

Question?